Real Options, Timber Insurance, and  
the Generalized Faustmann Formula under Risk and Uncertainty

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Approved for publication by the director of the Louisiana Agricultural Experiment Station as manuscript no. 20-22-xxxxx. Research supported in part by the National Institute of Food and Agriculture, U.S. Department of Agriculture, McIntire Stennis State Cooperative Research Project LAB 94315.

*Abstract*

Timber production is a risky business, with both price uncertainties resulting from fluctuations of stumpage prices and volume risk resulting from natural and man-made disasters. Past studies of reservation prices have been shown to be the equivalent of a real option. Harvesting decisions based on the reservation price produce results which are financially superior to that of decisions made under the assumption of certainty. Real options, however, are only meaningful if it can be delivered when called upon. When disasters strike, the underlying asset is destroyed and can no longer be delivered, thus making the real option irrelevant. In this paper, we will show that the price uncertainties can be managed with a real option while the volume risks can be addressed through timber insurance. Under such arrangement, forest management once again becomes a decision-making process under a certainty equivalent environment.

*Introduction*

Timber production is inherently a risky business. On the one hand, there is the price uncertainty resulting from stumpage price fluctuations. On the other hand, there is the stand volume risk caused by natural and man-made disasters. Over the years, it has remained one of the key topics in forest management and economic.

To overcome the stumpage price uncertainty, Brazee and Mendelsohn (1988) developed the reservation price method to determine both the value of the land and trees as well as when the stand should be harvested. Over the last 30 years, increasingly sophisticated mathematical tools have been employed to address forest management under price uncertainty. A partial list would include Alvarez and Koskela (2007) and the reference therein, Chladná (2007), Gadow (2000), Hughes (2000), Insley (2002), Insley and Rollins (2005), Morck et al. (1989), Rakotoarison and Loisel (2017), Sødal (2002), and Thomson (1992). All of them tried to internalize the effects of price uncertainty. Yousefpour et al (2012) provided a comprehensive review of the relevant literature in the context of dealing with risks and uncertainties caused by climate changes.

In the meantime, the simpler yet equally potent generalized Pressler formula (Chang and Deegen 2011) representing the first order condition of maximizing the generalized Faustmann formula has enabled forest owners/ managers to internalize the price uncertainty in management decisions. Within their framework, they suggested that a down-side risk tolerance level of about 75% is appropriate. Later, employing the growth and yield model used in Brazee and Mendelsohn (1988), Zhang and Chang (2018) found out that a down-side risk tolerance level of 95% would be appropriate based on the real sawtimber stumpage price for Louisiana from 1956 to 2015 with a mean of $169.19 /mbf and a standard deviation of $65.73/mbf.

Such approaches still face another challenge. As Plantinga (1998) pointed out the reservation price of Brazee and Mendelsohn (1988) represents the option value of timber harvesting under price uncertainty, unlike financial options, however, the underlying timber asset could be destroyed by man-made or natural disasters such as fire and hurricanes. For example, forest fires since 2018 in California and major hurricanes such as Florence in North Carolina in 2018, Michael in both Florida and Georgia in 2018, as well as Laura in 2020 and Ida in 2021 in Louisiana have clearly shown such risks. Once the underlying asset is destroyed, the option mentioned above is rendered worthless.

Volume risk has long been recognized. Early publications include the risk of fire by Martell (1980), Routledge (1980) and Reed (1984,1987). These studies were followed by Reed and Apaloo (1991), Yin and Newman (1996), Englin et al (2000), Stainback and Alavalapati (2004), Yoder (2004), Amacher (2005) and Loisel (2011, 2014) and Susaeta et al (2014). Recently, the generalized Pressler formula (Susaeta 2018) for the generalized Reed model (Susaeta et al. 2016) has been utilized to internalize the stand volume risk in management decisions. Except Xu et al. (2016), all the articles mentioned above assume that only one catastrophe may occur over the planned rotation. Most recently, in a rare application, Susaeta and Gong (2019) combined the reservation price approach with the Reed model to internalize both price uncertainty and volume risk.

This article takes a different approach. Instead, both the stumpage price uncertainty and timber volume risk will be outsourced; the former with an American put option and the latter with a standing timber property insurance to recreate a certainty equivalent environment for decision making.

Conceptually, the American put option enables the forestland owners/managers to have the choice but not the obligation of selling a specified amount of timber at a set price by a particular date. Such a purchase, thus, eliminates the price uncertainty over that time span. One obvious solution to overcome the volume risk is to purchase an annual property insurance for standing timber, like a home insurance, to protect the underlying asset. With a put option and a property insurance, the forestland owner would have created a certainty equivalent environment to determine the optimal harvest age.

*Incorporation of an American put option in the generalized Faustmann formula*

From the basic generalized Faustmann formula

(1)

Where

V1(t1) =P1(t1)Q1(t1) represents the stumpage value of a t1-year old stand per acre with P1(t1) being the stumpage price of the t1-year old stand and Q1(t1) being the stand volume per acre of the t1-year old stand.

A1,s represents the miscellaneous annual income or expenses per acre during the first rotation for year s, 0 < s < t1.

C1 represents the regeneration cost per acre of the first rotation.

r1 represents the annual interest rate during the first rotation.

LEV2 represents the land expectation value per acre at the end of the first rotation and the beginning of the second rotation.

To ensure that the forestland owner could sell the t1-year old timber stand at a specific price, conceptually, the forestland owner could purchase a put option to accomplish that goal. In financial terms, the purchaser of the put option has the choice to sell a given quantity of an item at a specified price by an expiration date. An American option allows the purchaser of the option to exercise it anytime between the time of purchase and its expiration time. A European option, on the other hand, can only be exercised at its expiration time. To apply this concept to forest management with price uncertainty, conceptually the forestland owner could purchase an American put option O(P1(T1), Q1(T1), τ1) at age τ1 with the specified price of P1(T1) for a specific quantity Q1(T1) by the expiration age T1, T1≥ t1 andτ1 < t1. For example, the put option could be purchased at age τ1=15, for an expiration age of T1=40 while the harvest age t1 could be somewhere between 15 and 40. To provide enough time to realize the benefit of the put option, T1 should be longer than the optimal rotation age under certainty. But since the longer the T1-τ1 is, the more expensive the put option would be. Further, the higher the specified price P1(T1) and the larger the specified volume Q1(T1), the costlier the put option would be. Therefore, judicious selection of these variables for the put option represents important management choices.

Incorporating the put option into equation (1), the generalized Faustmann equation becomes

(2)

Analytically, since O1(P1(T1), Q1(T1), τ1) causes a reduction in V1(t1), it behaves like a harvest tax. As a result, the optimal harvest age will be lengthened. By implication, the stumpage price uncertainty would lengthen the optimal harvest age. Further, the earlier the option is purchased, the lager the stumpage value reduction would be, the more pronounced the rotation age will be lengthened.

Ever since the publication of Brazee and Mendelsohn (1988), all the articles following that paper have implicitly assumed that the forest stand faces no risk. Recent events mentioned above, however, clearly demonstrated otherwise. Unlike financial instruments which face little or almost no quantity risk, significant acreages of forest stands can be destroyed natural disasters, such as fires and hurricanes as well as insects and diseases problems. Due to its long growth period, if the underlying forest asset is destroyed along the way, the put option can no longer be exercised and becomes meaningless.

One way to address this problem is to purchase an insurance policy to protect against the risk of losing the underlying standing timber to either natural or man-made disasters like the purchase of a home insurance. With such an insurance to protect the value of the standing timber every year, the insured amount increases over time as the timber stand ages. For simplicity, the insurance premium would equal a fixed percentage of the insured value of the standing timber. As such, the timber insurance acts like a timber tax annually on the value of the standing timber (Chang 1982, 2018). Let the annual insurance premium be φ% of the value of the standing timber. With the incorporation of both the put option and the timber insurance the generalized Faustmann formula would become

(3)

The standing timber insurance, therefore, acts as an increase in the discount rate, and would shorten the optimal harvest age. By implication, the volume risk would tend to shorten the harvest age. The higher the volume risk, the higher the insurance premium and the more the rotation age will shorten. The annual insurance premium also sheds interesting lights on the calculation of the land expectation value. For example, with the average 30-year inflation indexed Treasury at around 3%, a timber investment discount rate of 6% implies an annual insurance premium of 3% for timber investments. Most REITs, when calculating the value of their oversea investments often uses even higher discount rate, reflecting or implying a higher risk of oversea investments. Obviously, some countries are safer with a small to zero extra premium. On the other hand, timber investment in some other countries with higher perceived political, economic and/or financial risks justifiably face much higher discount rate.

*Empirical Application and Comparison*

In this section, we will compare of effects of applying three different approaches on the average optimal rotation age, and the land expectation value. The three approaches are that of Brazee and Mendelsohn (1988), Zhang and Chang (2018) as well as the put option approach developed above.

As shown in Table 1, with P(T1) = $169.19/MBF, and a standard deviation of $65.73/MBF (Zhang and Chang 2018), Q(T1) = 49.009MBF/acre at age 41 and an interest rate of 4%, the option value calculated by the Cox et al. (1979) method shows that at age 15 the put option is valued at $59.52 per MBF and gradually declines to $0 per MBF at age 41. For any year ω­1, ≤ ω­1 ≤ , the reservation price RP(ω­1) can be calculated as follows:

RP(ω­1) = + (4)

For example, given the average stumpage price and its standard deviation mentioned above, with an annual volume increment of nearly 25% at age 15, it will take a spot price higher than $726.28/MBF, literally with no upside potential and 100% downside risk to justify its harvest. At age 25, with an annual volume increment of nearly 8.5%, the spot price must exceed $297.96/MBF with less than 2.6% upside potential and over 97.4% downside risk to justify its harvest.

To examine the impact of the put option on the optimal harvest age without the inclusion of the timber insurance, the results of three different approaches – Brazee and Mendelsohn’s (1988) reservation price, the reservation price through a put option obtained in this paper, and the fixed downside risk approach of Zhang and Chang (2018) were compared.

The results of the simulations with a land expectation value at the beginning of the second rotation is assumed to be $1200/acre. are shown in Table 2. The results by the Brazee and Mendelsohn (1988) method and that of the put option approach reported in this paper are quite similar in both the average harvest age, its distribution, and the average land expectation value. The approach by Zhang and Chang (2018), on the other hand, with a fixed downside risk threshold and upside potential

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Please run the comparisons between the three approaches - Brazee and Mendelsohn, Zhang and Chang and the put option approach reported in this paper and provide their optimal rotation ages and their LEV1. The attached spreadsheet has the reservation prices by Brazee and Mendelsohn and the put option.

*Discussion*

At present, the option market does not exist for standing timber stumpage. The reservation price shown in Table 1 calculated with the put option essentially provides a virtual framework to establish a price sensitive timber harvesting policy that depends on the age and growth rate of the stand as well as the average stumpage price and its standard deviation. Should the real option become a reality, a further opportunity emerges. When the timber stand is harvested before the expiration date of the put option, the forestland owner/manager could sell both the standing timber at a price above the reservation price and at the same time sell the put option to recover a portion of the purchase cost of the put option. For example, at age 15, the 49 MBF of put option would cost $2,916.48. If the stand were harvested at age 25 at a stumpage price above $296.96 per MBF, the put option at that time could be sold at $55.95 per MBF for a total value of $2,742.04 to recover a portion of the initial purchase cost.

Furthermore, it should be noted that with the put option forest owners/managers are no longer passive timber price takers. Instead, by specifying their target price in the put option, they would now be active price setters. Its social welfare implications could be profound and far reaching as Gong and Löfgren (2007) have shown.

According to Averill and Frost (1933), forest fire insurance and by extension standing timber insurance has been offered to private forestland owners as early as 1902 in Denmark, 1912 in Norway, 1914 in Finland, and 1919 in Sweden. Furthermore, around the world, such insurances are now available in China, Japan, Chile, New Zealand, South Africa, and France (Zhang and Stenger 2014). In the United States, in 1917, Timberlands Mutual Fire Insurance Company began underwriting for private forestland owners in New Hampshire but unfortunately, only managed to stay in business for a brief 8 months (Williams 1949). Since then, despite attempts by two chiefs of the Forest Service (Graves 1919, Pinchot 1919), a textbook (Matthews 1935) and efforts by researchers such as Sparhawk (1920), Herbert (1928), Averill and Frost (1935), as well as Shephard (1937, 1939) on experience grading and rate setting, culminating with a major review by Williams (1949) of such efforts, forest insurance business had attracted limited business interests. Between 1950 and 2000 scientific literature on forest insurance went into a hiatus. Since 2000, timber insurance has been discussed in the literature (see, for example, Brunette and Couture 2008, 2013; Brunette et al. 2014a, 2014b, 2015; Chen, Goodwin, and Prestemon 2014, Holecy and Hanewinkel 2006, 2014; Sauter et al. 2016, Zhang and Stenger 2014). In the past, high insurance premium (Heske 1938), either perceived or real, has been the main reason that the forest insurance has not been widely purchased in both Germany and the United States. In recent years, at least two private companies have been underwriting timberland insurance in the U.S. South. Hopefully, as the insurance risk is diversified geographically and over a much larger land base, a positive cycle would ensue. the insurance premium would drop and be affordable to an even larger number of forestland owners.

*Conclusions*

In this paper, real options and timberland insurance have been incorporated into the generalized Faustmann formula to recreate a certainty equivalent environment. This represents a new opening in analyzing the impact of risk and uncertainty in timber management. Many aspects of this new formula need to be explored. Empirically, for example, the age to purchase the real option τ and the expiration age of the real option T should be investigated. Furthermore, the target price of the option needs to be carefully examined to explore both its implication of social welfare as well as risk taking behavior. Theoretically, the incorporation of the prices of multiple products such as pulpwood, chip-and-saw as well as sawtimber into the real option needs to be explored.

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Table 1. Reservation prices determined by the Brazee and Mendelsohn method and the put option method.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Ave. price = | 169.19 | STD= | 65.73 |
|  | Brazee and Mendelsohn | | Put option based | |
| t | Reservation Price | Downside Risk | Reservation Price | Downside Risk |
| 15 | 688.28 | 100.00% | 726.28 | 100.00% |
| 16 | 586.54 | 100.00% | 614.96 | 100.00% |
| 17 | 512.36 | 100.00% | 535.00 | 100.00% |
| 18 | 456.84 | 100.00% | 475.73 | 100.00% |
| 19 | 414.39 | 99.99% | 430.59 | 100.00% |
| 20 | 381.37 | 99.94% | 395.42 | 99.97% |
| 21 | 355.32 | 99.77% | 367.42 | 99.87% |
| 22 | 334.51 | 99.40% | 344.69 | 99.62% |
| 23 | 317.67 | 98.81% | 325.95 | 99.15% |
| 24 | 303.88 | 97.98% | 310.28 | 98.41% |
| 25 | 292.42 | 96.96% | 296.96 | 97.40% |
| 26 | 282.78 | 95.80% | 285.71 | 96.19% |
| 27 | 274.52 | 94.55% | 276.06 | 94.80% |
| 28 | 267.34 | 93.23% | 267.58 | 93.28% |
| 29 | 260.97 | 91.87% | 260.03 | 91.65% |
| 30 | 255.20 | 90.47% | 253.19 | 89.94% |
| 31 | 249.84 | 89.01% | 246.91 | 88.15% |
| 32 | 244.71 | 87.47% | 241.05 | 86.29% |
| 33 | 239.65 | 85.81% | 235.49 | 84.34% |
| 34 | 234.46 | 83.96% | 230.19 | 82.33% |
| 35 | 228.89 | 81.81% | 225.04 | 80.23% |
| 36 | 222.60 | 79.18% | 219.86 | 77.96% |
| 37 | 215.06 | 75.74% | 214.45 | 75.45% |
| 38 | 205.29 | 70.86% | 208.59 | 72.55% |
| 39 | 191.19 | 63.11% | 201.79 | 69.00% |
| 40 | 167.88 | 49.21% | 192.86 | 64.06% |
| 41 | 169.19 | 50.00% | 169.19 | 50.00% |

Figure 1: Reservation prices determined by the Brazee and Mendelsohn method and the put option method.